

Check the following text for flow. What is the new information and what is the old information in each sentence? What is the logical subject of each sentence? Does each sentence logically lead to the next? Improve for readability by restructuring sentences or inserting additional phrases or sentences or choosing notation if appropriate.

If the text raises some questions for you as a reader, formulate these questions.

Angles in Four Dimensions

Life in 4 dimensions is different from 3 dimensions. There are two angles between two two-dimensional subspaces through the origin, which we will from now on simply call *planes*, not just one. Let P_1 be the projection to the first plane π_1 and let P_2 be the projection to the second plane π_2 . The matrix $P_1P_2P_1$ does the following: First project to π_1 , then project to π_2 , then project to π_1 . The directions perpendicular to π_1 are annihilated by the first step. Therefore, two eigenvalues are 0. The projection P_1 leaves a vector in π_1 unchanged, then P_2 maps it to π_2 . then P_1 maps it back to π_1 . Vectors from P_1 are thus mapped to P_1 by the matrix $P_1P_2P_1$, and it is a self-adjoint operator. Therefore it has two real eigenvalues with perpendicular eigenvectors in P_1 , and these eigenvalues are $\cos^2 \alpha_1$ and $\cos^2 \alpha_2$, where α_1 and α_2 are the angles between π_1 and π_2 . Project the unit circle in P_1 to P_2 . The result will be a (possibly) degenerate ellipse. We denote its major and minor axis by $\cos \alpha_1$ and $\cos \alpha_2$, because the angle between the point of P_1 that is mapped to the major axis forms an angle α_1 between itself and its image, and similarly for α_2 . The role of these angles is to fix the relative position between two planes up to congruence.