Abstract.

We give an O(n log n) time algorithm for the exact congruence testing problem for two point sets in 4-space. This problem is to decide if two point sets in the 4-dimensional Euclidean space are the same up to rotations and translations. Although the problem is sensitive for numerical errors, since matching point sets within small errors are shown to be NP-hard, we restrict our concerns to the exact case using the Real Random-Access Machine (Real-RAM) model.

For two and three dimensional cases, there are known O(n log n) algorithms due to Manacher (1976) and Atkinson (1987). It has been conjectured that O(n log n) algorithms should exist for any dimensions. Although there have been a series of results for proving this conjectured bound, the best known algorithm by Brass and Knauer (2000) achieves O(n^ceil(d/3) log n) in d-dimensional space, yet far from the conjecture and therefore O(n^2 log n) in 4-space.

To establish the new algorithm, the paper makes use of properties of planes in 4-space, such as angles, distances, and packing numbers, and also properties of rotations in 4-space. In particular, the paper provides an alternative construction of Hopf fibration.