An sequence of points on the graph of a strictly concave function will be called an *arc*. As usually, the size of an arc is the number of points in it.

Let $r$ be a fixed number. An *$r$-chain [with corners] with $k$ arcs* consists of $k$ arcs of size $r+1$, the rightmost point of the $i$th arc ($1 \le i \le k-1$) being the leftmost point of the $(i+1)$th arc, so that any three points are in upward position if and only if they belong to the same arc. Three points are in upward position if they form a clockwise oriented triangle when sorted by $x$-coordinate.

An $r$-chain with $k$ arcs will be denoted by $\mathrm{CH}(r,k)$. The number of points in $\mathrm{CH}(r,k)$ is $rk+1$. In particular, points on the graph of a convex function form a 1-chain.

One can construct an $r$-chain with $k$ arcs as follows:

- Take $k+1$ points $p_{1,0}, p_{2,0}, p_{3,0}, \ldots, p_{k+1,0}$ (sorted by $x$-coordinate) in downward position;
- For each $i = 1, 2, \ldots, k$ add $r-1$ points $p_{i,1}, p_{i,2}, p_{i,3}, \ldots, p_{i,r-1}$ (sorted by $x$-coordinate) on the segment $p_{i,0}p_{i+1,0}$;
- Replace the segments $p_{i,0}p_{i+1,0}$ by upward convex circular arcs of sufficiently big radius, while keeping points on these arcs and moving them vertically upwards, so that the orientation of triples of points that did not lie on the same segment is not changed.

The points $p_{1,0}, p_{2,0}, p_{3,0}, \ldots, p_{k+1,0}$ will be called *corners*. The $j$th corner $p_{j,0}$ will be also denoted by $V_j$. See Figure 1 for an example.

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Let $C$ be a segment of $M$ such that $C \triangleleft A$ (such a segment exists by Proposition 1). Since $A$ is the minimum element of $M_B^L$, we have $C \in M_B^R$, that is, $B \triangleleft C$.

If $C = D$ then we have $D \triangleleft A, A \triangleleft B, B \triangleleft D$: that is, the relation $\triangleleft$ in the triple $\{A, B, D\}$ is not linear; therefore $\{A, B, D\}$ is of type C.

Suppose now that $C \ne D$, and consider the matching $\{A, B, C, D\}$. We have $C \triangleleft A, A \triangleleft B, B \triangleleft C$. So, the relation $\triangleleft$ in the matching $\{A, B, C, D\}$ is not linear; therefore, $\{A, B, C, D\}$ is of type C. Now, by Proposition 2, some segment in $\{A, B, C, D\}$ must be bigger than $D$ with respect to $\triangleleft$. Since $B \triangleleft D$ and $C \triangleleft D$, we have $D \triangleleft A$. So, we have $D \triangleleft A, A \triangleleft B, B \triangleleft D$, and this means (as above) that $\{A, B, D\}$ is of type C.