§2. An Exercise on Technical Writing

In the following excerpt from a term paper, N denotes the nonnegative integers, N^n denotes the set of n-tuples of nonnegative integers, and $A_n = \{(a_1, \ldots, a_n) \in N^n \mid a_1 \geq \cdots \geq a_n\}$. If $C, P \subset N^n$, then $L(C, P)$ is defined to be $\{c + p_1 + \cdots + p_m \mid c \in C,\ m \geq 0$, and $p_j \in P$ for $1 \leq j \leq m\}$. We want to prove that $L(C, P) \subseteq A_n$ implies $C, P \subseteq A_n$.

The following proof, directly quoted from a sophomore term paper, is mathematically correct (except for a minor slip) but stylistically atrocious:

$L(C, P) \subset A_n$
$C \subset L \Rightarrow C \subset A_n$
Spse $p \in P,\ p \notin A_n \Rightarrow p_i < p_j$ for $i < j$
$c + p \in L \subset A_n$
$\therefore\ c_i + p_i \geq c_j + p_j$ but $c_i \geq c_j \geq 0, p_j \geq p_i \because (c_i - c_j) \geq (p_j - p_i)$
but $\exists$ a constant $k \ni c + kp \notin A_n$
let $k = (c_i - c_j) + 1 \qquad c + kp \in L \subset A_n$
$\therefore\ c_i + kp_i \geq c_j + kp_j \Rightarrow (c_i - c_j) \geq k(p_j - p_i)$
$\Rightarrow k - 1 \geq k \cdot m \qquad k, m \geq 1 \qquad$ Contradiction
$\therefore\ p \in A_n$
$\therefore\ L(C, P) \subset A_n \Rightarrow C, P \subset A_n$ and the
lemma is true.

A possible way to improve the quality of the writing:

Let N denote the set of nonnegative integers, and let

$$N^n = \{\, (b_1, \ldots, b_n) \mid b_i \in N \text{ for } 1 \leq i \leq n \,\}$$

be the set of n-dimensional vectors with nonnegative integer components. We shall be especially interested in the subset of "nonincreasing" vectors,

$$A_n = \{(a_1, \ldots, a_n) \in N^n \mid a_1 \geq \cdots \geq a_n\}\,. \tag{1}$$

If C and P are subsets of N^n, let

$$L(C, P) = \{c + p_1 + \cdots + p_m \mid c \in C, m \geq 0, \text{ and } p_j \in P \text{ for } 1 \leq j \leq m\} \tag{2}$$

be the smallest subset of N^n that contains C and is closed under the addition of elements of P. Since A_n is closed under addition, $L(C, P)$ will be a subset of A_n whenever C and P are both contained in A_n. We can also prove the converse of this statement.

Lemma 1. *If $L(C, P) \subseteq A_n$ and $C \neq \emptyset$, then $C \subseteq A_n$ and $P \subseteq A_n$.*

Proof. (Now it's your turn to write it up beautifully.)